

Full one-loop electroweak radiative corrections to single photon production in e^+e^-

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Abstract

Large scale calculation for the radiative corrections required for the current and future collider experiments can be done automatically using the GRACE-LOOP system. Here several results for $e^+e^- \rightarrow 3$ -body processes are presented including $e^+e^- \rightarrow e^+e^-H$ and $e^+e^- \rightarrow \nu\bar{\nu}\gamma$.

1 Introduction

The research of the Higgs particle is one of the most important subjects in the particle physics. Its discovery is never be the final target but the detailed study of its property is the key to understand the standard model and its possible extension. The same refers to expected supersymmetric(SUSY) particles. Experimental study will be done in the future e^+e^- linear colliders(LC). In order to improve the theoretical prediction for the experimental data with high accuracy, the electroweak(EW) radiative corrections are required for various signal channels and major background ones.

Armored by the automated systems, now the calculation is available not only for the two-to-two processes but for the two-to-three processes. Already, the analyses of the following three processes are found in the literature.

The important Higgs production processes in LC are the Higgsstrahlung $e^+e^- \rightarrow ZH$ and the W -fusion process $e^+e^- \rightarrow \nu_e\bar{\nu}_eH$. The latter is dominant in the large parameter space.

The tree level calculation shows that the W -fusion is dominant at $W = 800\text{GeV}$ for $M_H \leq 600\text{GeV}$ and also even at $W = 500\text{GeV}$ for Higgs in the mass-range preferred by precision data study[1]. The full EW correction for this process is done in [2] and has demonstrated the importance of the complete calculation. The resulted weak correction is $-2 \sim -4\%$ in G_μ -scheme.

The direct study of Higgs Yukawa coupling can be done in LC for the production channel $e^+e^- \rightarrow t\bar{t}H$ whose cross section is order of fb for a light Higgs for $W = 700 \sim 1000\text{GeV}$. The QCD correction has been studied[3] and the correction is negative in the high energy region. The full EW correction is studied in [4] and has shown that the correction is similar magnitude as that of QCD and is positive in the high energy region.

More challenging item is the study of Higgs self coupling (Higgs potential). Though the yield is low, 0.2fb for $W \simeq 500\text{GeV}$ and $M_H \simeq 120\text{GeV}$, the study based on detailed simulation[6] predicts $\sim 10\%$ precision will be expected. This requires the estimation of the radiative correction. The genuine weak cor-

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rection is a few % for the probable values of W, M_H [5].

In this paper, preliminary results for the two processes, $e^+e^- \rightarrow e^+e^-H$ and $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ are presented based on the works by the authors.

2 System

The calculation is done using the automated system for the perturbative calculation, GRACE-LOOP. The detailed description is found in [7] and only a few features are given here.

2.1 5-point functions

In order to calculate the radiative correction for two-to-three processes, one needs 5-point functions. Since a set of 5 vectors are linearly dependent in 4-dimensional space, an identity for the Gram determinant leads to the reduction of a 5-point function into a sum of lower point functions[8]. The explicit implementation of the reduction formula is not unique. For example, see [9] for the similar implementation by which the one-loop QED correction is calculated for $e^+e^- \rightarrow \mu^+\mu^-H$. We have examined several methods and found that the following is appropriate in order to make the produced symbolic formulae *relatively* short.

$$I_5 = \int \frac{N}{D_0 D_1 D_2 D_3 D_4} = \sum G_{\mu\nu\cdots\sigma} \int \frac{\ell^\mu \ell^\nu \cdots \ell^\sigma}{D_0 D_1 D_2 D_3 D_4} \quad (1)$$

where the numerator is a rank M tensor of ℓ , $D_0 = \ell^2 + X_0$ and $D_j = \ell^2 + 2r_j\ell + X_j$ ($j = 1, 2, 3, 4$). We can define the 'metric tensor' by

$$g^{\mu\nu} = \sum_{ij} r_i^\mu (A^{-1})_{ij} r_j^\nu, \quad A_{ij} = r_i r_j \quad (2)$$

to obtain the identity

$$\ell^\mu = g^{\mu\nu} \ell_\nu = \sum_{ij} \frac{1}{2} r_i^\mu (A^{-1})_{ij} (D_j - D_0 + X_0 - X_j). \quad (3)$$

Substituting the above identity into Eq.(1), the numerator becomes

$$N = \sum_{\alpha=0}^4 E_\alpha(\ell) D_\alpha + F. \quad (4)$$

The first term of Eq.(4) turns to be box integrals whose numerator is a rank $M-1$ tensor of ℓ . The second term of Eq.(4) becomes a scalar 5-point function. This can be reduced by the identity

$$1 = \sum_{\alpha=0}^4 (a_\alpha + b_\alpha r_j \ell) D_\alpha \quad (5)$$

which can be obtained from $D_0 - X_0 = \ell_\mu g^{\mu\nu} \ell_\nu$.

Table 1: The number of Feynman diagrams

process	tree	1-loop
$e^+e^- \rightarrow \nu\bar{\nu}H$	12(2)	1350(249)
$e^+e^- \rightarrow t\bar{t}H$	12(6)	2327(758)
$e^+e^- \rightarrow ZHH$	27(6)	5417(1597)
$e^+e^- \rightarrow e^+e^-H$	42(2)	4470(510)
$e^+e^- \rightarrow \nu\bar{\nu}\gamma$	10(5)	1099(331)

full set(production set)

2.2 Non-linear gauge

We have implemented the non-linear gauge fixing defined by the following Lagrangian[10, 7].

$$L_{GF} = -\frac{1}{\xi_w} F^+ F^- - \frac{1}{2\xi_z} (F^Z)^2 - \frac{1}{2\xi} (F^A)^2 \quad (6)$$

$$F^\pm = \left(\partial^\mu \mp ie\tilde{\alpha} A^\mu \mp i\frac{ec_W}{s_W} \tilde{\beta} Z^\mu \right) W_\mu^\pm + \xi_w \left(M_W \chi^\pm + \frac{e}{2s_W} \tilde{\delta} H \chi^\pm \pm i\frac{e}{2s_W} \tilde{\kappa} \chi_3 \chi^\pm \right) \quad (7)$$

$$F^Z = \partial^\mu Z_\mu + \xi_z \left(M_Z \chi_3 + \frac{e}{2s_W c_W} \tilde{\epsilon} H \chi_3 \right) \quad (8)$$

$$F^A = \partial^\mu A_\mu \quad (9)$$

The main reason we use this gauge fixing is to check the output of the system. In the calculation based on an automated system, the diagnostic stage is highly important. Among

many check items, e.g., renormalizability, infrared stability, etc., the gauge invariance is most powerful check.

In order to keep the loop integral simple and stable, the numerator of vector propagators is to be $g^{\mu\nu}$. Under this restriction, the gauge check is not possible in the conventional R_ξ gauge. Sometimes, the non-linear gauge fixing was used to reduce the number of diagrams: For instance, the choice of $\tilde{\alpha} = 1$ removes $\gamma W \chi$ vertex. We take $\xi_w = \xi_z = \xi = 1$ while $\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\epsilon}, \tilde{\kappa}$ are arbitrary. Then the number of diagrams is larger than that in the linear gauge fixing.

GRACE system generates all possible Feynman diagrams for a specified process. We call them 'full set'. Using the full set and in quadruple precision we repeat the computation of the cross section at a few phase space points for a different set of numerical values of gauge parameters. When they agree within reasonable digits, we can confirm the performance of the system. Then we discard the diagrams containing scalar-electron couplings to define 'production set'. The cross section of the production set is integrated in the phase space in double precision (sometimes in quadruple precision for the check). The numbers of diagrams are shown in Table 1. For the 1-loop, the counter terms are counted also.

3 Results

3.1 $e^+e^- \rightarrow e^+e^-H$

This channel is associated with the dominant Higgs production processes $e^+e^- \rightarrow \nu_e \bar{\nu}_e H$. Higgsstrahlung $e^+e^- \rightarrow ZH$ and the Z -fusion process contribute to this process. As is shown in Fig.1, the tree cross section rises quickly after the ZH threshold and has the values of $O(10)\text{fb}$.

The parameter set for the calculation is as follows: $M_W = 80.3766\text{GeV}$, $M_Z = 91.1876\text{GeV}$, $\Gamma_Z = 2.4956\text{GeV}$, $M_H = 120\text{GeV}$, $m_t = 174\text{GeV}$, $W = 200 \sim 3000\text{GeV}$, $k_{cut} = 0.05E$. The width of Z only appears at resonant poles.

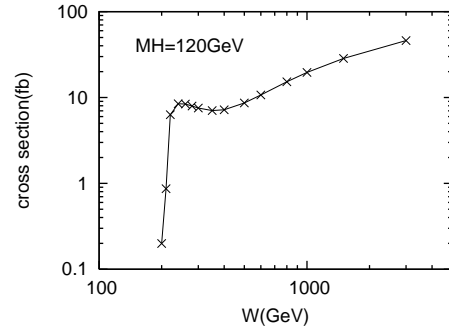


Figure 1: Tree cross section for $e^+e^- \rightarrow e^+e^-H$.

In Fig.2, the EW correction is shown. Here, $\delta(W)$ is the fraction of the deviation from the tree cross section after subtracting the well-known QED correction. For the parameter set used here, $\Delta r = 2.55\%$. In the so-called G_μ -scheme, $\delta(W)^G = \delta(W) - 3\Delta r$. G_μ -scheme explains the major part of EW correction in lower energy region, but in higher energy region.

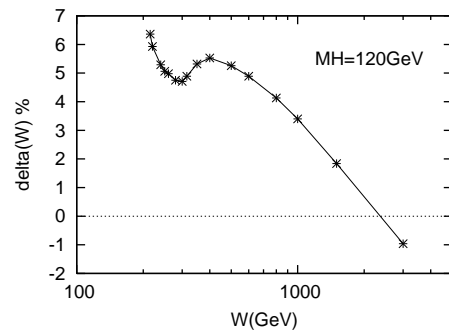


Figure 2: Weak correction $\delta(W)$ for $e^+e^- \rightarrow e^+e^-H$.

3.2 $e^+e^- \rightarrow \nu \bar{\nu} \gamma$

The single photon signal contributes to many SUSY particle study. The precise understanding of the radiative correction in the standard model is essential for such a study. Also this channel serves for the neutrino counting. The physical parameters are the same as for $e^+e^- \rightarrow e^+e^-H$. For the radiated photons, we have applied the so-called OPAL cut, i.e., $p_T(\gamma) > 0.05E$, $15^\circ < \theta(\gamma) < 165^\circ$.

As is shown in Fig.3, the tree cross section

is about 1fb and the ν_e -contribution dominates for $W > 500\text{GeV}$.

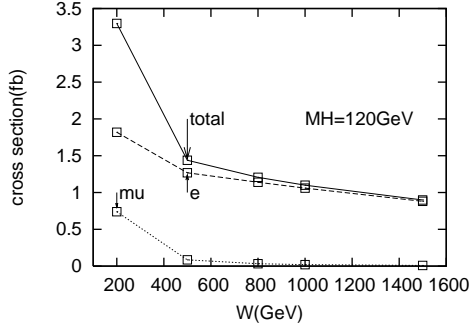


Figure 3: Tree cross section for $e^+e^- \rightarrow \nu\bar{\nu}\gamma$.

The energy dependence of EW correction differs between ν_e -contribution and that of ν_μ . For instance, at $W = 1.5\text{TeV}$ the correction for ν_μ is 4 times larger than that of ν_e . However, as ν_μ term is small in high energy, the weak correction is determined mostly by the ν_e -contribution. The correction is shown in Fig.4. The definition of the $\delta(W)$ is the same as in the last subsection. The energy dependence shows common structure to other processes. The photon energy and angle distributions will be presented in the coming publication.

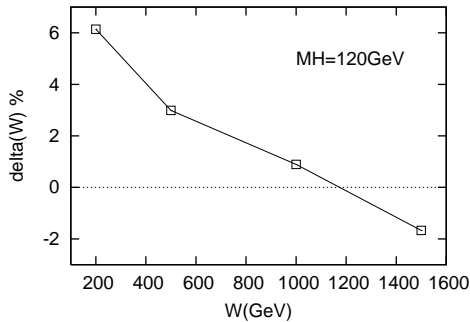


Figure 4: Weak correction $\delta(W)$ for $e^+e^- \rightarrow \nu\bar{\nu}\gamma$.

4 Final remarks

The study of higher order effects is important for the future study of Higgs and SUSY particles. It has been demonstrated that the GRACE-LOOP system serves well for such a

study. We have calculated the full EW radiative corrections for two processes, $e^+e^- \rightarrow e^+e^-H$ and $e^+e^- \rightarrow \nu\bar{\nu}\gamma$. At $W = 500\text{GeV}$ and for $M_H = 120\text{GeV}$, the genuine EW correction is 5% for the former and 3% for the latter. The presented results are preliminary, and the detailed study will appear in the forthcoming publications.

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